International Journal of Theoretical Physics, Vol. 42, No. 8, August 2003 (© 2003)

Coulomb Blockade of a Mesoscopic RLC Circuit

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Received March 3, 2003

The quantum theory of the mesoscopic RLC circuit and the condition for Coulomb blockade are given by using canonical quantization and a unitary transformation from the classical equation of motion. Our results show that there is a threshold voltage ε_{T} in the circuit. The threshold voltage is related not only to the junction capacitance and inductance, but also to the resistance of the circuit. Generally speaking, the larger the resistance, the larger the threshold voltage. This clarifies the phenomenon of the Coulomb blockade of the dissipative mesoscopic circuit.

KEY WORDS: dissipative mesoscopic RLC circuit; coulomb blockade; threshold voltage.

1. INTRODUCTION

Mesoscopic physics deals with the frontiers between classical and quantum physics. Phenomena such as persistent current, Coulomb blockade, magnetoresistance fluctuations, and other, are currently studied in this area (Allahverdyan and Nieuwenhuizen, 2002; Flores, 2002). With the rapid development of nanophysics and nanoelectronics, the size of electric devices are gradually becoming smaller and, in the future, are expected to approach nanometer scale. Mesoscopic expressions for electric devices such as capacitance and inductance are formulated. When the scale of fabricated electric materials reaches a characteristic dimension, namely, Fermi wavelength, the quantum mechanical properties of mesoscopic physics become important, since the charge carriers such as electrons exhibit quantum properties while the application of classical mechanics fails. Many researches in the quantum effect of the mesoscopic circuit have been done (Fan and Pan,

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0020-7748/03/0800-1765/0 © 2003 Plenum Publishing Corporation

1998; Ji and Lei, 2002; Wang *et al.*, 2001; Zhang *et al.*, 1998; Zhang *et al.*, 2001).

Very recently, new effects have been predicted to arise in ultrasmall tunnel junctions with capacitance C, such as the charging energy of a single electron $e^2/2C$, which exceeds the characteristic energy $k_B T$ of thermal fluctuations. Simple energy considerations suggest that the tunneling of a single electron is completely blocked when the junction capacitor holds a charge less than $e/2$. A large amount of experimental data exists that seems to support the theoretical ideas of the Coulomb blockade of tunneling (Li and Chen, 1996a,b; Wang *et al.*, 2000; Yu and Liu, 1998). Undoubtedly, quantum effects of a mesoscopic circuit must be considered in the tunneling process of electrons where capacitors are considered tunnel junction. Ji *et al.* has studied the effect caused by a single electron crossing a tunnel, and the results show that when an electron crosses a mesoscopic capacitor that is linked by atoms, because of the effect of Coulomb force there exists Coulomb Gap (Ji *et al.*, 2002c). That is to say: when the external voltage is lower than certain threshold voltage V_T, the tunnel current equals zero because of the effect of Coulomb force; when the external voltage is much higher than the threshold voltage V_T, the tunnel current is in direct proportion to the external voltage; with an external constant current source supply, the circuit will undergo Single Electron Tunnel Oscillation (SET Oscillation).

Based on the discreteness of electric charge, Li and Chen *et al.* have studied the quantum fluctuations of the charge and current in a nondissipative mesoscopic circuit (Li and Chen, 1996a,b). Their results show that the Coulomb blockade exists, and point out that the Coulomb blockade is relative not only to the junction capacitance, but also to inductance of the circuit. The electromotive force ε only takes discrete values under adiabatic approximation. However, they do not consider the influence of dissipation on the Coulomb blockade of the capacitance tunnel junction.

In this paper, we study the quantum mechanical effects of a dissipative mesoscopic RLC circuit by using canonical quantization and a unitary transformation from the classical equation of motion. We discuss the influence of dissipation on the Coulomb blockade. Our results are in good agreement with the experimental data.

2. DIAGONALIZING THE HAMILTONIAN

The classical kinetic equation of a RLC circuit is

$$L\ddot{q} + R\dot{q} + C^{-1}q = \varepsilon(t), \tag{1}$$

where L, C, and R are the inductance, capacitance, and resistance, respectively. $\varepsilon(t)$ is the external voltage source. $q(t)$ is charge that is regarded as a coordinate of the system. The generalized current is defined as $p(t) = L\dot{q}$. As everyone knows, the resistance of a circuit is the result of collisions between crystal lattice and electrons. And the crystal lattice oscillation is equivalent to the movement of phonons. So, in fact, the mesoscopic RLC circuit can be equal to an interactive system of an electromagnetic harmonic oscillator and the environment bath of the lattice oscillators (Ji *et al.*, 2002d). In this work, it is easy to confirm that the coordinate $\hat{q}$ and momentum $\hat{p}$ of a mesoscopic dissipative circuit are a pair of linear Hermitian operators and satisfy the commutation relation $[\hat{q}, \hat{p}] = i\hbar$. In the present work, therefore, following the standard quantization principle, we quantize $\hat{q}$ and $\hat{p}$. We assume that the following commutation relation is valid

$$[\hat{q}, \hat{p}] = i\hbar. \tag{2}$$

We consider the adiabatic approximation so that $\varepsilon(t)$ is considered as a constant ε. The Hamiltonian of this system is

$$\hat{H}_0 = \frac{1}{2L}\hat{p}^2 + \frac{1}{2C}\hat{q}^2 + \beta(\hat{p}\hat{q} + \hat{q}\hat{p}) + \hat{q}\varepsilon. \tag{3}$$

$\beta = R/L$. One can check that Eq. (3) is a Hermitian operator. We introduce the unitary transformation

$$\hat{S} = \exp\left\{i\frac{R}{2\hbar}\hat{q}^2\right\}, \tag{4}$$

and use the formula

$$e^{\lambda\hat{A}}\hat{B}e^{-\lambda\hat{A}} = \hat{B} + \lambda[\hat{A}, \hat{B}] + \frac{1}{2}\lambda^2[\hat{A}, [\hat{A}, \hat{B}]] + \cdots\cdot$$

From Eqs. (3) and (4), we get

$$\hat{H} = \frac{1}{2L}\hat{p}^2 + \frac{1}{2}L\omega^2\hat{q}^2 + \frac{1}{2C}\hat{q}^2 + \hat{q}\varepsilon, \tag{5}$$

where

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}. \tag{6}$$

Now $\hat{H}$ represents the harmonic oscillator exactly. In the previous work, most authors considered the electric charge continuous. As a matter of fact, the electric charge is discrete and this must play an important role in the theory of quantized mesoscopic circuits. To take account of the discreteness of electric charge, we impose that the eigenvalues of the self-adjoint operator q take discrete values, namely

$$\hat{q}|q\rangle = mq_{\mathrm{e}}|q\rangle, \tag{7}$$

where $m \in z$ (set of integers), $q_{\mathrm{e}} = 1.602 \times 10^{-19}\ C$, the elementary electric charge; $|q\rangle$ stands for eigenstates of electric charge for the circuit. So we can define minimum shift operators

$$\hat{Q} = \exp\left(i\frac{q_e}{\hbar}\hat{p}\right),$$

from which the following commutation relations follow: $[\hat{q}, \hat{Q}] = -q_e\hat{Q}$, $[\hat{q}, \hat{Q}^+] = q_e\hat{Q}^+$, $\hat{Q}\hat{Q}^+ = \hat{Q}^+\hat{Q} = 1$. These relations can determine the structure of the whole Fock space. Similar to the known results, we can define the right and left discrete derivative operators as follows:

$$\nabla_{q_e} = \frac{\hat{Q} - 1}{q_e}, \qquad \bar{\nabla}_{q_e} = \frac{1 - \hat{Q}^+}{q_e}.$$

Obviously $\nabla_{q_e}^+ = -\bar{\nabla}_{q_e}$. Consequently, one can obtain the self-adjoint "momentum" operators

$$\hat{p} = -i\frac{\hbar}{2}[\nabla_{q_e} + \bar{\nabla}_{q_e}] = -\frac{\hbar}{2q_e}(\hat{Q} - \hat{Q}^+)$$

Regarding the discreteness of electric charge, the Schrodinger equation for a dissipative mesoscopic circuit takes the form

$$\left[-\frac{\hbar^2}{2q_e^2L}(\hat{Q} + \hat{Q}^+ - 2) + \frac{1}{2}L\omega^2\left(q + \frac{\varepsilon}{L\omega^2}\right)^2\right]|\Psi\rangle = \left(E + \frac{\varepsilon^2}{2L\omega^2}\right)|\Psi\rangle. \quad (8)$$

3. COULOMB BLOCKADE

Due to the discreteness of electric charge, Eq. (7) must hold: i.e.

$$\varepsilon = mq_eL\omega^2, \quad (9)$$

for m an integer. Equation (9) suggests that there exists the Coulomb blockade in a mesoscopic dissipative RLC circuit. We can see that the electromotive force ε only takes discrete values, given by Eq. (9). The discrete value of the external voltage reflects the physical nature of the Coulomb blockade in a mesoscopic circuit and the phenomenon of the Coulomb blockade produced by the quantization of charges. Supposing capacitance and inductance are both constants, from Eq. (9), the electromotive force ε is only dependent on the size of the resistance. When $m = 1$, the external voltage is

$$\varepsilon_T = q_e\left(\frac{R^2}{L} - \frac{1}{C}\right), \quad (10)$$

ε_T is a defined threshold voltage that is caused by the system Coulomb force. It indicates that there must exist an original threshold voltage ε_T to overcome the Coulomb force when the electrons transfer through the junction of the capacitor.

And if the external voltage is lower than the original threshold voltage, the current crossing junction equals zero, thus, Coulomb blockade phenomenon appears.

The threshold voltage (or Coulomb Gap) is relative not only to the junction capacitance and inductance, but also to resistance of the circuit. The results indicate that the resistances in the mesoscopic circuit play an extremely important role in the observation of the Coulomb blockade. Generally speaking, the larger the resistance, the larger the threshold voltage is, with the phenomenon of the Coulomb blockade of the dissipative mesoscopic circuit clearing up. When $R^2 = LC$, the phenomenon of the Coulomb blockade disappears. There is no doubt that the phenomenon of the Coulomb blockade can be observed easier for an overdamping RLC circuit than underdamping. The resistance value is small; it is difficult to observe the Coulomb blockade.

4. DISCUSSION

According to the relations (Li and Chen, 1996a,b)

$$\langle p'|(\nabla_{q_e} - \bar{\nabla}_{q_e})|p\rangle = \frac{4\pi\hbar}{q_e^2}\left[\cos\left(\frac{q_e}{\hbar}p\right) - 1\right]\cdot\delta(p - p'),$$

$$\langle p'|\hat{q}^2|p\rangle = -\frac{2\pi\hbar^3}{q_e}\frac{\partial^2}{\partial p^2}\cdot\delta(p - p').$$

The Schrodinger Eq. (8) in the p representation is

$$\left\{-\frac{\hbar^2}{q_e^2 L_1}\left[\cos\left(\frac{q_e}{\hbar}p\right) - 1\right] - \frac{\hbar^2}{2}\left(\frac{1}{C} - \frac{R^2}{L}\right)\right\}|\tilde{\Psi}(p)\rangle = \left(E + \frac{\varepsilon^2}{2L\omega^2}\right)|\tilde{\Psi}(p)\rangle$$

The solution of Mathieu equation above is

$$\tilde{\psi}_l^+(p') = ce_l\left(\frac{\pi}{2} \pm \frac{q_e}{2\hbar}p',\ k\right),$$

or

$$\tilde{\psi}_{l+1}^-(p') = se_{l+1}\left(\frac{\pi}{2} \pm \frac{q_e}{2\hbar}p',\ k\right).$$

where $k = (\frac{2\hbar}{q_e^2 L\omega})^2$; "+" and "−" denote the even and odd parity solutions, respectively. Here $Ce_l(z,\ k)$ and $Se_l(z,\ k)$ are periodic Mathieu functions. In this case, there exist infinitely many eigenvalues $\{a_l\}$ and $\{b_l\}$ ($l = 0, 1, 2, \ldots$) that are not identically equal to zero. Then, the energy spectrum is

$$\tilde{E}_l^+ = \frac{q_e^2}{8}\left(\frac{1}{C} - \frac{R^2}{L}\right)a_l(k) + \frac{\hbar^2}{q_e^2 L} - \frac{\epsilon^2}{2L\omega^2}$$

$$\tilde{E}^-_{l+1} = \frac{q_e^2}{8}\left(\frac{1}{C} - \frac{R^2}{L}\right) b_{l+1}(k) + \frac{\hbar^2}{q_e^2 L} - \frac{\epsilon^2}{2L\omega^2}.$$

In consideration of the conditions $k \ll 1$, the quantum fluctuations of electric current are calculated as following expression in the ground state

$$\langle p \rangle = 0. \tag{11}$$

$$\langle p^2 \rangle = \frac{\hbar^2}{2q_e^2}\left[1 - \frac{3}{2}\cdot\left(\frac{\hbar^2}{q_e^4 L\omega}\right)^2 + \cdots\right]. \tag{12}$$

Equation (11) indicates that the average value of the current is zero whether the circuit is in the ground state. And Eq. (12) shows that there exists current quantum zero point fluctuation.

5. CONCLUSION

In the above discussion, we studied the quantization of a dissipative mesoscopic RLC circuit. Taking the charge's discreteness into account, we proposed a quantum theory for the mesoscopic electric circuit. The Schrodinger equation for coupling circuit can become the well-known Mathieu equation. The Coulomb blockade has been addressed in this mesoscopic dissipative circuit. There is no question that the phenomenon of the Coulomb blockade can be more easily observed for an overdamping than an underdamping RLC circuit.

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